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Investigation of Crack in Beam Structure using an Adaptive-Genetic Algorithm (AGA)

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Abstract

Fault detection and continuous condition monitoring in structural and machine elements are very sensitive topics and gaining significant value as a current research area. Due to the continuous loading and unloading of these elements, fatigue occurs. For the above-mentioned reason, crack is initiated and propagated. The initiation of any type of crack changes the physical properties of the structural and machine elements, which directly affects the lifetime of the element. The presence of any discontinuity changes the physical properties of the element, which also changes elastic properties. These alterations in physical properties change the modal properties of the structural elements. These changes in the vibration criteria can be used for the identification and quantification of the damage. In this research work, the vibration parameters are combined with Artificial Intelligence (AI) to predict the damage location. Here the natural evolution-based Genetic Algorithm (GA) has been used for the training of vibration features (frequencies). It has been discovered that the original AI methods are sometimes not able to give the proper prediction of damage location as they may be trapped in the local optimum. So, to counteract this loophole and to make it more flexible so that it can adjust to the constraints of real-life problems, a data mining method using Regression Analysis (RA) has been proposed and the results have been compared.

Author Keywords. Damage. Ncl. Ncd. Regression Analysis.

Type: Research Article

1. Introduction

With the passage of time, damage detection in machine and structural elements is attaining a significant amount of importance. Damage or fault detection is one of the methods of continuous condition monitoring. Damage in structural elements occurs due to cracks. It is known to all that the structural and machine elements are continuously loaded and unloaded. Due to the continuous loading and unloading process, these parts undergo fatigue. Fatigue is the main reason for the occurrence of discontinuity (cracks) in the elements. The discontinuities usually change the physical properties of the elements. The only aim of the researchers is to find the stress distribution and the strain analysis according to the stress distribution. Various research works also depict different analytical methods used for damage detection.

The research works of Dziedziech et al. (2016), Feng and Feng (2016), and Sun et al. (2016) depict some of the analytical methods used for damage detection. The appearance of any minute crack can affect the stiffness of the element. The stiffness of any material has a direct impact on the modal features of the material. Measured modal parameters may be used to predict the approximate crack size. It becomes difficult to find the damage location only by using natural frequencies, but the use of mode shape enables to get a better solution rather than natural frequencies. But this paper is made with the use of natural frequencies for simplicity. The methodologies can be direct and inverse methods for damage detection. In the direct methods, the natural frequencies and mode shapes can be calculated using crack geometry and analytical method. But most of the natural frequencies do not give proper location rather than mode shapes. But as the number of variables is increased, the algorithm is stalked in the search space, which will be trapped in local solution. From the above discussion, it can be noticed that researchers should be more focused on extracting more sensible vibration signals and using advanced artificial intelligence techniques. These new methods use modern signal processing techniques combined with AI techniques. These methods can be used as a robust tool for online damage detection.

In this work, an inverse engineering method has been applied for the diagnosis of damage in a cracked structural element and this is a trial method to propose an online method, as mentioned in the above paragraph. In this paper, the first three natural frequencies are collected from finite element analysis (FEA). Some of the researchers have also used finite element analysis (FEA) to get the vibration responses at the earliest to build a search space. In this article, a minute hairline transverse crack has been considered. For simplicity of analysis, a V-type crack is considered for finite element analysis.

There are works of different researchers who have used different Artificial Intelligence (AI) and soft computing methods to train the data pool. Some researchers (Ranjbaran and Ranjbaran 2014; Quila, Mondal, and Sarkar 2014) have also used finite element analysis (FEA) to get the vibration responses at the earliest to build a search space. Discrete values for the geometry of the crack are taken to find the natural frequencies of the first three modes. There are works by different researchers, mainly Baviskar and Tungikar (2013), Tiachacht et al. (2018), and Alexandrino, Gomes, and Cunha (2020), have used different Artificial Intelligence (AI) and soft computing methods to train the data pool. Though different evolutionary algorithms use different coding systems and many of them use numerical values also. Here the numerical values are converted into dimensionless values. This is done by comparing the values of cracked structural elements and uncracked structural elements. Then these dimensionless values of the input and output variables are trained in soft computing methods (evolutionary algorithm). Here Genetic Algorithm (GA) has been used. Genetic Algorithm (GA) application is not new; it has been noticed that Genetic Algorithm (GA) has been applied by different researchers in various fields. It has also been successfully applied in different engineering works for optimization and prediction. But it has been observed by different researchers that most of the time, the Genetic Algorithm (GA) tends to get trapped in local solutions. So, to get a global solution, various methods have been accomplished with a Genetic Algorithm (GA). Different researchers have used various statistical methods to get a perfect solution from the Genetic Algorithm (GA) as Shahidi et al. (2015), Lazarevic, Kanapady, and Kamath (2004) and Gres et al. (2017), and to find out the severities of cracks. Though Genetic Algorithm (GA) has been successfully applied by the above-mentioned researchers, it has some shortcomings, like trapping of the algorithm in local algorithms. There are many reasons for stalling of the algorithm due to the trapping of the algorithm. One of the reasons may be due to the presence of residual error in the data acquisition method. So, in this article, a new method has been proposed. Here, a statistical method has been used with a Genetic Algorithm (GA). Regression Analysis (RA) can be used as a statistical method to treat the data collected from different methods (Chun et al. 2020) and locate the damage. The association of the statistical method, i.e., Regression Analysis (RA), makes the Genetic Algorithm (GA) more adaptive, which can be used for different problems with different constraints.

Leon-Medina et al. (2021) implemented the manifold learning algorithm to classify structural damage. The applications of Laplacian's Eigen map, stochastic proximity embedding along with locally linear embedding algorithms were considered as the proposed manifold algorithm. The proposed methods include the normalization of data, reduction of dimensionality, isomap and the machine learning model of K-Nearest Neighbors with the cross-validation of the data set. The proposed method was validated with experimental data and found to be very appreciable and useful for structural damage detection. Pacheco-Chérrez, Cárdenas, and Probst (2021) applied the De-noising and wavelet transformation approach to find out and determine the dimension and orientation of crack-type damage in composite structures. Jena and Parhi (2021) also developed a crack detection procedure in the domain of RNNs for moving load dynamic problem. They have also verified the method with FEA.

Khalkar et al. (2022a) have carried out some studies on mode shape-based vibration analysis to investigate the geometry of cracks in a cantilever beam. Later again, Khalkar et al. (2022b), in their next issue, performed numerical and experimental studies to discuss the different types of crack shapes (V and U shapes) on the applicability of damage percentage on a damped cantilever beam. Vakil-Baghmisheh et al. (2008) applied Genetic Algorithm (GA) as a fault diagnosis approach to identify cracks in beam-like structures.

As far as the literature review report is concerned, so many researchers have done their work in Genetic Algorithms (GAs), ANNs, Fuzzy logic and other different types of methods/algorithms for crack detection in structure. But the crack detection method in the domain of the data mining approach is very scanty. In this work, an inverse engineering method has been applied for the diagnosis of damage in a cracked structural element and this is a trial method to propose an online method as mentioned. In this paper, the first three natural frequencies are collected from finite element analysis (FEA) and treated as input to the proposed Genetic Algorithm (GA) model. In this proposed method, Regression Analysis (RA) has been combined with a Genetic Algorithm (GA) to get rid of local solution and locate the normalized crack location (ncl) and normalized crack depth (ncd).

2. Finite Element Analysis for Damaged Structure

In this analysis, the beam is considered an Euler-Bernoulli beam (Figure 1) of a consistent rectangular cross area for free bending vibration and having a single crack:

$$EI\frac{d^4y}{dx^4} - m_b\omega_i^2 y = 0 \tag{1}$$

m_b- is the mass of the beam per unit length (kg/m)

ω_{ib}- is the natural frequency of the i^{th} mode (rad/sec)

E-is the modulus of elasticity (N/m^2)

I-is the moment of inertia (m^4)

By characterizing $\alpha_b{}^4 = \frac{m_b\omega_i^2}{EI}$ mathematical statement is adjusted to the following equation:

$$\frac{d^4y}{dx^4} - \alpha_b{}^4 y = 0 \tag{2}$$

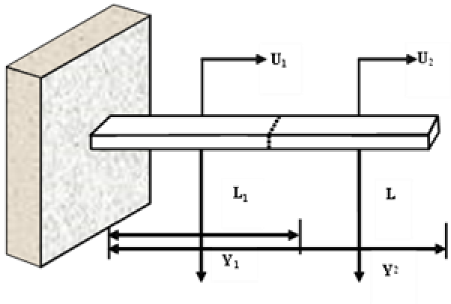

Figure 1: Cracked cantilever beam with

The solution for the above equation is

$$y=Pcos\alpha_i x+Qsin\alpha_i x+Rcosh\alpha_i x+Ssinh\alpha_i x \quad (3)$$

Where P, Q, R, S are constants and 'α_i' is a frequency parameter.

The differential equation for the system is given as:

$$[m_b]\ddot{x} + [c]\dot{x} + [k]x=fsin(\omega_b t) \quad (4)$$

$$[m_b]\ddot{x} + [k]x=0 \quad (5)$$

$$\{x\} = \{\phi\}Sin\omega_b t \quad (6)$$

Where

ϕ - The Eigen vector or mode shape for the vibrating beam

ω_b - Natural frequency for the vibrating beam

$$\omega_b{}^2[m_b]\{\phi\}\sin\omega_b t + [k]\{\phi\}\sin\omega_b t = 0 \quad (7)$$

After simplification, it becomes

$$([k] - \omega_b{}^2[m_b])\{\phi\} = 0 \quad (8)$$

$$[a_b\text{-}\alpha_b I]x=0 \quad (9)$$

a_b - Square matrix

α_b-Eigen values

I - Identity matrix

x- Eigen vector

In modal analysis of structures Eigen equation is written in terms of k, m_b, and ω_b with $\omega_b{}^2=\lambda$.

There are two possible solutions for Equation (8)

1. If $\left|([k]\text{-}\omega_b{}^2[m_b])\{\phi\}\right| \neq 0$ is an insignificant solution where $\{\phi\} = 0$
2. If $([k]\text{-}\omega_b{}^2[m_b])\{\phi\} = 0$ is a significant solution where $\{\phi\} \neq 0$

$$|([k] - \omega_b{}^2[m_b])| = 0 \quad (10)$$

$$|([k] - \lambda[m_b])| = 0 \quad (11)$$

The determinant is zero only at discrete Eigen values

$$|([k] - \omega_b{}^2[m_b])|\{\phi_i\} = 0 \; i = 1,2,3,\ldots. \quad (12)$$

$$[k^e] = \int [B_b(x)]^T EI[B_b(x)]dx \quad (13)$$

$$[B_b(x)] = \{H_1{}''(x)H_2{}''(x)H_3{}''(x)H_4{}''(x)\} \quad (14)$$

H (x) is known as a type of interpolation function used to find the stiffness matrix.

The above-mentioned method has been used to find the natural frequencies of the faulty structural elements. These data are used to generate a search place or data pool.

3. Investigation of Damage using Genetic Algorithm

In the last two decades, the topic of fault detection has gained considerable attention in different engineering fields like mechanical, civil, and aerospace engineering. The existence of the crack introduces deviations in the dynamic properties of the structure. These deviations in the bending beam can be used for health monitoring of the structures. The actual structural damage is nonlinear in nature and most of the literatures available have considered it as linear type. It is assumed that the cracks impart some of the nonlinearity characteristic to the vibration responses of the cracked structure that can be used for damage detection. But all the methods based on the dynamic responses of the structure are not appropriate for online health monitoring of the structural elements. In recent years many researchers have applied upcoming AI techniques for fault detection in any type of structural and machine elements. In this paper, one of the potent evolutionary algorithms has been addressed for damage detection. An algorithm based on the Genetic Algorithm (GA) has been designed and developed to be put in the field of fault detection.

As the Genetic Algorithm (GA) is one of the best evolutionary algorithms, it has been successfully applied by different researchers. It can be applied to an optimization problem with different encoding systems. Many researchers have used it to solve multidimensional, non-differential and non-continuous problems. Genetic Algorithms (GAs) can be easily transferred to existing simulations and models. Though due to the presence of the above-mentioned advantages, there are some loopholes in Genetic Algorithms (GAs). Its application in controls that are performed in real-time is limited because of random solutions and convergence, in other words, this means that the entire population is improving, but this could not be said for an individual within this population. So, to overcome these drawbacks, Regression Analysis (RA) has been used.

During data collection, we always face residual errors. In this paper, the data collected from the dynamic and experimental analysis are treated in the Genetic Algorithm (GA) to obtain the approximate crack geometry. During the collection of data, some human inaccuracy is surely added to the data collected; this human error may be in the form of residual error. If, by some technique, these residual errors can be reduced or omitted.

3.1. Representation scheme used in Genetic Algorithm

Genetic Algorithms (GAs) are optimization algorithms. It is well known that each search-based optimization algorithm requires a depiction scheme that makes an effort to present a solution to a problem. The cryptography used for the Genetic Algorithm (GA) method depends on the user. The regular types of cryptography used in Genetic Algorithms (GAs) are binary encoding, permutation encoding, value encoding, tree encoding, etc. In the depicted research work binary type of cryptography has been used for the Genetic Algorithm (GA) (as shown in Figure 2).

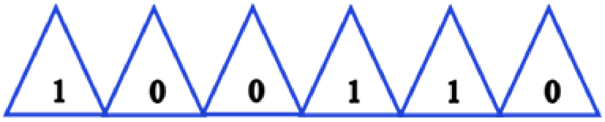

Figure 2: Binary presentation of a chromosome

3.2. Parent selection

The operation selection is one of the genetic operators used to pick parents for reproduction operation. Different ways of selecting the parents for crossover are like roulette wheel tournament selection, selection, rank selection and some others. In the present research work, the parents are selected according to their fitness numbers. The fitness ranking for parents handpicked is done by training the solutions or individuals on an objective function/ fitness function.

3.3. Fitness function

For selecting the parents on fitness basis, fitness function is generated. It is used to rank the solutions in the solution space. The fitness numbers are then used to choose powerful solutions through natural selection.

3.4. Crossover operation

The genetic operation named Crossover (Figure 3) is a very important operator whose presence makes them different from other search-based optimization methods.

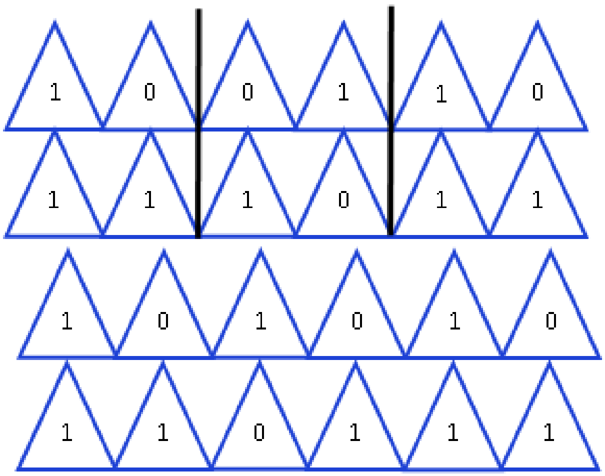

Figure 3: Presentation of crossover

The best individuals are selected using natural selection method using any of the above stated selection methods. For crossover operation to proceed, a crossover site should be selected according to the problem defined. During the crossover operation, the part of chromosome succeeding and prior to the crossover point is swapped and copied among the parents. From the crossover operation two new off springs are generated and the fitness numbers of the off springs are calculated using fitness function. Then the fitness numbers of the parents and off springs are compared and the individual with most effective solutions are added in the search space, so that solution space capacity will be constant.

3.5. Mutation operation

This has been observed by different researchers that due to repeated crossover operation, the algorithm stalls and does not give a better solution. So, at that time Mutation operation is introduced. During this operation some portion or bits are changed within the problem friendly range. Mutation operation is used to add diversity in the population. It can intercept the algorithm to be trapped in a local minimum. The Mutation operation sometimes leads to total changeover in individuals, so the amount of mutation in the algorithm is always kept small, within the range 0.001 and 0.01. Figure 4 depicts the change in bits due to mutation operation.

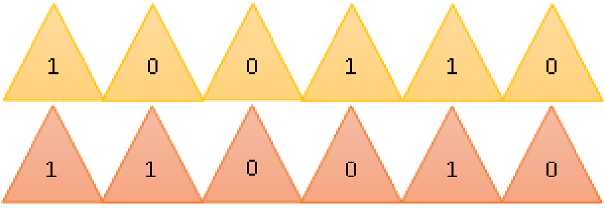

Figure 4: Schematic presentation of mutation operation

4. Genetic Algorithm Steps

1. The dependent and independent variables and fitness function are defined first.
2. The function to find out the fitness numbers of the individuals is given in equation

$$\textit{Fitness function/Objective function}=\sqrt{\left(nfnf_{fld}-nfnf_{1-k}\right)^2+\left(nsnf_{fld}-nsnf_{1-k}\right)^2+\left(ntnf_{fld}-ntnf_{1-k}\right)^2} \quad (15)$$

$nfnf_{fld}$ = Normalized First natural frequency of the field

$nfnf_{1-k}$ = Normalized first natural frequency

$nsnf_{fld}$ =Normalized Second natural frequency of the field

$nsnf_{1-k}$ = Normalized second natural frequency

$ntnf_{fld}$ =Normalized Third natural frequency of the field

$ntnf_{1-k}$ = Normalized third natural frequency

k = iterations numbers

A solution space having twelve numbers of solutions is generated. This solution space is generated using finite element analysis.

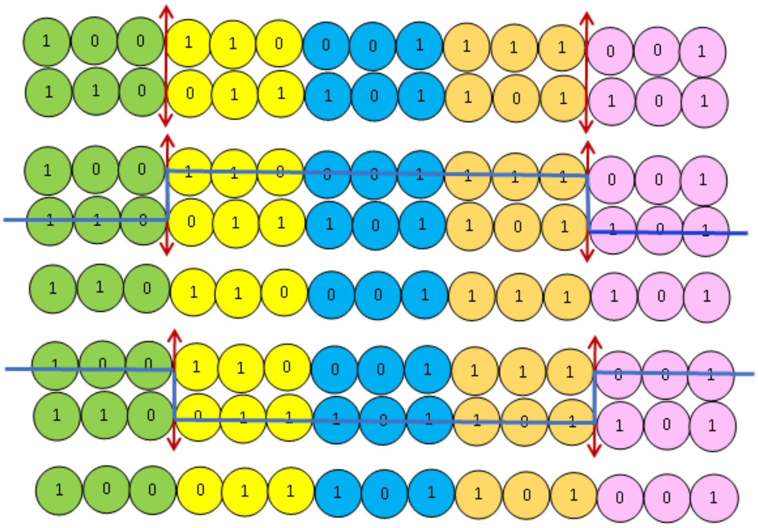
Figure 5: Schematic presentation of a two-point crossover

3. Based on the fitness numbers, two parents are decided from the solution space.
4. After the selection of parents, they undergo crossover. In this work, a two-point site crossover is used. Each chromosome for this problem contains five variables (dependent and independent), and each variable contains three bits, so the chromosome comprises fifteen bits. The crossover sites are decided by three bits left of the chromosome and

three bits right of the individuals. Figure 5 show the depiction scheme of parent with crossover sites.

5. The children from the crossover of the parent chromosomes are found. Figure 6 depicts the implementation of a two-point location of crossover points in the current Genetic Algorithm (GA) for the investigation of faults in the structural elements.

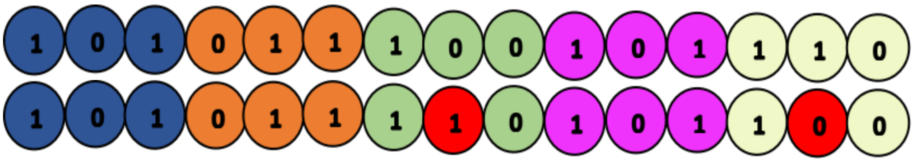

Figure 6: Presentation of mutation

6. Only crossover may trap the algorithm in the local optimization. A binary encoding has been adapted in the current research paper. So, to avoid this situation, mutation has been introduced in the algorithm. The mutation rate used is always kept very small, as stated in the previous section; here, it is kept as 0.1% for the current problem. As the chromosome consists of 15 bits, only two bits at a time are flipped or altered.
7. After the crossover and mutation process, fitness function is again applied to all the parents and offsprings. The fitness values of the parents and offspring are then compared. Then the best fit member is added to the solution space for further calculations. If a parent comes as the best fit, then the desired output (ncd, ncl) is the output belonging to that set.
8. The iterations are till the algorithm meets the threshold values. The algorithm terminates when it meets the threshold values.

5. Application of Regression Analysis with Genetic Algorithm

All data acquisition systems consist of three essential elements-Sensor, Signal Conditioning and Analog-to-Digital Converter (ADC). In modern days sensors are used to collect data. The data may be temperature, force, and movement to voltage or current signals. Common sensors include thermocouples, thermistors, and RTDs to measure temperature, electrometers to measure movement, and strain gauges to measure force. Many times, signal conditioning is done within the transducers. The heart of the data acquisition system is the Analog-to-Digital Converter (ADC). This is a converter used to condition data to digital numbers. The data acquisition step of the structural health conditioning method requires reading from the sensors attached at different sites of the structural elements. The data collected from the sensors are accompanied by residual errors. So, filtering is the only option for clearing the errors. In this paper, for filtering and feature extraction Regression Analysis (RA) has been used. Figure 7 describes the data acquisition process. As discussed earlier, in the data collected from any method for data acquisition, many errors are also collected with residual errors. So, to minimize the inclusion of errors in the data collection, Regression Analysis (RA) has been added to the Genetic Algorithm (GA). First, the data is treated in the Genetic Algorithm (GA) and the results are produced. The data is trained in Regression Analysis (RA) and then Genetic Algorithm (GA).

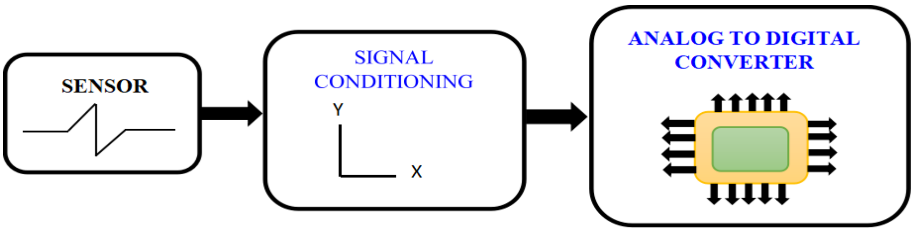

Figure 7: Presentation of data acquisition

6. Result and Discussions

In the present study, a crack detection approach has been developed and subsequently tested by considering a cracked cantilever beam. The developed method is the Adaptive-Genetic Algorithm (AGA). The forward problem has been analyzed in FEA method. The reverse problem has been analyzed in the Adaptive-Genetic Algorithm (AGA) method. Here the Genetic Algorithm (GA) and Regression Analysis (RA) methods are comprised. In order to avoid errors in the generation of data like in Fuzzy Logic System (FLS), Regression Analysis (RA) is utilized here. To verify the proposed algorithm, a numerical example was formulated with a single cracked cantilever beam at different normalized crack locations and depths. The size of the cracked beam considered for the analysis is 100×5×0.4cm. The structural element considered here is a cantilever beam. Five sets of examples are considered and verified with the proposed technique. The normalized crack depth (ncd) is the ratio of the thickness of the crack to the actual thickness of the beam, while the normalized crack location (ncl) is the ratio of the location of the crack from the fixed end to the actual length of the beam. The five sets of examples are shown in Table 1 and Table 2.

The first three normalized natural frequencies are considered as the input to the proposed model, whereas the ncd&ncl are considered as the output of the model for training as well as the testing process. 85% of the data set is used for training, while the remaining 15% of the data set is for testing. The results obtained from the approach are compared with FEA and shown in Table 1 and Table 2.

Sl. No	nfnf	nsnf	ntnf	ncd	ncl	ncd using the GA technique	ncl using the GA technique	percentage error ncd	percentage error ncl
1	0.9922	0.9913	0.9968	0.322	0.2184	0.3076	0.2084	4.46	4.55
2	0.9932	0.9927	0.9977	0.3	0.205	0.2865	0.1958	4.49	4.45
3	0.9946	0.9942	0.9972	0.2872	0.231	0.2741	0.2206	4.55	4.48
4	0.9959	0.99772	0.999	0.122	0.2185	0.1164	0.2086	4.54	4.5
5	0.9974	0.9977	0.9965	0.274	0.3622	0.2617	0.3459	4.48	4.49

Table 1: Results from Genetic Algorithm

Sl. No	nfnf	nsnf	ntnf	ncd	ncl	rcd using the FLS technique	rcl using the FLS technique	percentage error rcd	percentage error rcl
1	0.9922	0.9913	0.9968	0.322	0.2184	0.3122	0.2117	3.04	3.03
2	0.9932	0.9927	0.9977	0.3	0.205	0.291	0.1988	3	2.99
3	0.9946	0.9942	0.9972	0.2872	0.231	0.2785	0.2240	3	3.01
4	0.9959	0.99772	0.999	0.122	0.2185	0.1183	0.2119	3.01	2.99
5	0.9974	0.9977	0.9965	0.274	0.3622	0.2659	0.3514	2.95	2.98

Table 2: Results for Adaptive-Genetic Algorithm (AGA)

Table 1 and Table 2 give a detailed comparison of the results from Genetic Algorithm (GA) without data training and Genetic Algorithm (GA) with data training using Regression Analysis (RA). It has already been stated that the natural frequencies (first three) are normalized for data generation. The first five columns give the normalized or dimensionless values of the dependent and independent variables. Then the value of the error percentage is calculated using the equation given below.

$$\frac{(Data\ from\ FEA\text{-}Data\ from\ AGA)}{Data\ from\ FEA} \times 100 \tag{16}$$

The results from the standalone Genetic Algorithm (GA) and Adaptive-Genetic Algorithm (AGA) are compared. The calculated errors show the direct comparison between the methods and are found to be suitable for structural damage detection. This methodology has been done in a supervised manner.

7. Conclusion

In the current decade, vibration base methods are getting significant research importance. This research work tries to extend the research in this field by the application of Artificial Intelligence (AI) techniques. As mentioned earlier in this work, though a number of works have been done on Genetic Algorithms (GAs) in different fields and for damage detection, it has been observed that due to the data acquisition process for the training procedure, human errors are collected. Due to the involvement of residual errors, the result produced may not be proper. So, to reduce the amount of residual errors, Regression Analysis (RA) has been added to the Genetic Algorithm (GA). Nowadays, damage detection methods and techniques are gaining significant importance to avoid serious catastrophic failures. In this paper, one such method has been proposed for damage identification and location.

This method can be treated as one of the NDT methods based on the vibration signature of the cracked or damaged structural or machine element. The suggested method comprises of Genetic Algorithm (GA) and Regression Analysis (RA). As human errors are involved in (FLS) in generating data and making the rules, here, Regression Analysis (RA) is used to clear the errors in the data generation. It has clearly been observed that the result from the suggested technique gives superior results (3%) as compared to Genetic Algorithm (GA) (standalone process 4.5%). This proposed method can also be used to detect damage in multi-crack beams with different end conditions.

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