

MATLAB educational tool for teaching Rayleigh random variable

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Abstract

This paper presents the set of MATLAB demonstrations dedicated to teaching characteristics of Rayleigh random variable, which is important in wireless communication systems, radars, data science, and vibrations, among others. This educational tool is created as a complementary teaching tool in the online classes, during the COVID-19 pandemic, with the aim to help students to better understand the behavior and characteristics of this random variable. The Demo tool consists of three sets of demonstrations: Description, Parameters and Relation with Other Random Variables, Operations on Rayleigh Random Variable, and Generalization. The demo is made in MATLAB and utilizes all benefits provided by MATLAB, such as simplicity, good graphic environment, and interactive use, among others. Additionally, this demo is used with other demo programs, also made in MATLAB, in teaching/learning process of random variables. Students were very satisfied with this demo tool expressing that helped them understanding better the behavior, and characteristics of the Rayleigh random variable.

1. Introduction

The computer-based programs are important complementary tool in teaching/learning process, in many engineering subjects, ([Lyon and Magana 2020](#)). Our experience shows that teaching in a traditional way a primer course on random variables and processes is a little bit abstract and difficult for students ([Dolecek 2013](#)). In order to increase teaching efficiency, and help in better understanding random variables and processes, we created various computer-based demo programs, as a complement to the traditional classes in the classroom, as for example ([Dolecek 2012, 1997](#)). This educational tool, created for the online classes during the COVID pandemic, is devoted to a Rayleigh random variable (RV), which is important in wireless communication systems ([Rappaport 1996; Nguyen 2009](#)), data science ([Yuri 2012](#)), survival analysis ([Elviana 2019](#)), to model a statistical population with subpopulations ([Karim 2011](#)), ultrasound ([De la Cruz 2005](#)), and radars ([Fernandez 2016](#)), among others.

The principal motivation for this work is to help students in a better understanding of a Rayleigh RV by visualizing its characteristics, and demonstrating its relation with other RVs. To our best knowledge, there is no such educational tool presented in literature, dedicated to an extended and deep learning of Rayleigh RV.

The demo tool is made in MATLAB, which is widely accepted as software for research and teaching/learning at many universities across the world ([Aktas 2020](#)).

The principal features of this demo tool are:

- *Easy to use.* The user is guided during the presentation with the instructions what to do in next steps, and consequently he/she can be concentrated completely on the subject.
- *Interactivity.* In order to get an active participation of students, there is option that the users choose the parameters by their own.
- *Repeatability.* The demo can be repeated as many times, as one wants, with different parameters.
- *Easy installation.* The tool can be easily installed in any platform in the MATLAB environment.

The demo tool consists of three sets of demonstrations, described in details in continuation:

1. Description, Parameters and Relation with Other Random Variables.
2. Operations on Rayleigh Random Variable.
3. Generalization.

The rest of the paper is organised as follows. Next section describes the first set of demonstrations dedicated to the description of Rayleigh RV, and its relation with other RVs. Section 3 demonstrates some operations on Rayleigh RV. Finally, Section 4 introduces generalization of Rayleigh RV. Discussion is given in Section 5, while in Section 6 are presented Conclusions.

2. Description, Parameters, and Relation with Other Variables

This set of demonstrations has the purpose to present the generation and principal characteristics of Rayleigh RV, Probability Density Function (PDF), Distribution, Probability, and parameters: Mean value, Variance, Mode, and Median. The relation with other RVs is also included. The demonstrations are listed below:

- Generation, PDF, Distribution, Probability, and Parameters.
- Shape of PDF.
- Rayleigh and Rice Random Variables.
- Rayleigh and Uniform Random Variables.
- Rayleigh and Exponential Random Variables.
- Rayleigh and Chi-Square Random Variables.

All demonstrations are briefly described, and illustrated in continuation.

2.1. Generation, PDF, Distribution, Probability, and Parameters

In this demonstration is presented the generation of Rayleigh RV r by generating the normal RVs, x_1 and x_2 , both with zero mean value, and the variance σ_x^2 , chosen by user,

$$r = \sqrt{x_1^2 + x_2^2} \quad (1)$$

It is important to mention that in all demonstration the parameter of Rayleigh RV σ_x^2 is the variance of the normal RVs in (1) and not the variance of the Rayleigh RV. Next, the PDF and distribution are estimated and compared with the mathematical PDF and distribution. The mean value, variance, mode and median are related with the parameter σ_x^2 . As an example [Figure 1](#) illustrates the estimations of PDF and distribution, along with the mathematical PDF and distribution, for chosen parameter $\sigma_x^2=4$. The estimations of PDF and distribution are presented with the bar diagrams.

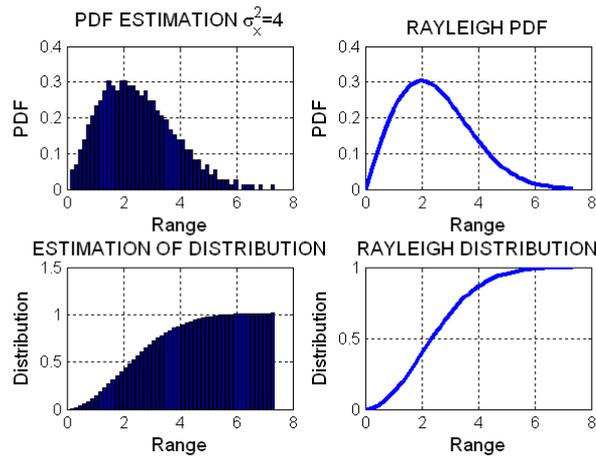


Figure 1: Estimation and mathematical PDF and distribution.

2.2. Shape of PDF

In this demonstration is shown how the shape of Rayleigh PDF depends on the parameter σ_x^2 . The user is invited to choose 4 parameters σ_x^2 in an increased order. Figure 2 shows the PDFs for chosen parameters: $\frac{1}{2}$, 1, 4, and 9.

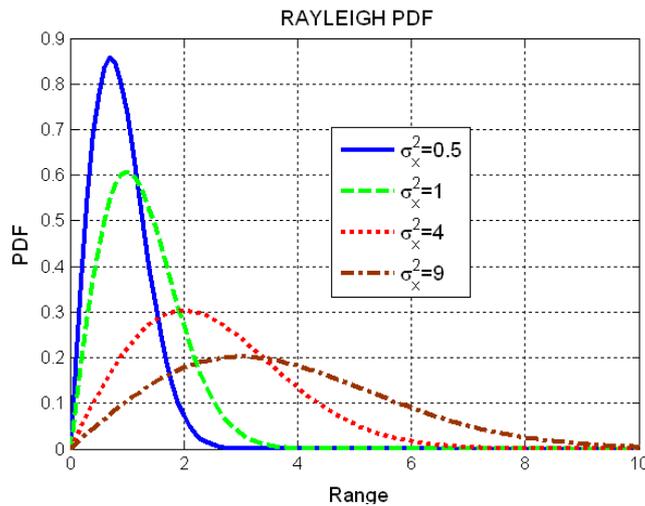


Figure 2: Shape of PDF.

2.3. Rayleigh and Rice Random Variables

The effect of a stationary (nonfading) signal arriving with many weaker multipath signals gives rise to the Rice RV signal (Rappaport 1996). As signal amplitude A, becomes weaker, the composite signal has an envelope that is Rayleigh variable r, while as signal amplitude A becomes dominant, the composite signal envelope approaches to a normal variable. Otherwise, the envelope is a Rice RV, r_r .

$$r_r = \sqrt{(x_1 + A)^2 + x_2^2}, \tag{2}$$

where x_1 and x_2 are normal random variables with zero mean and the variance equal to σ_x^2 .

The demonstration is illustrated in Figure 3, which presents the estimated PDFs for the values of $\sigma_x^2 = 1$, and different values of A , chosen by users.

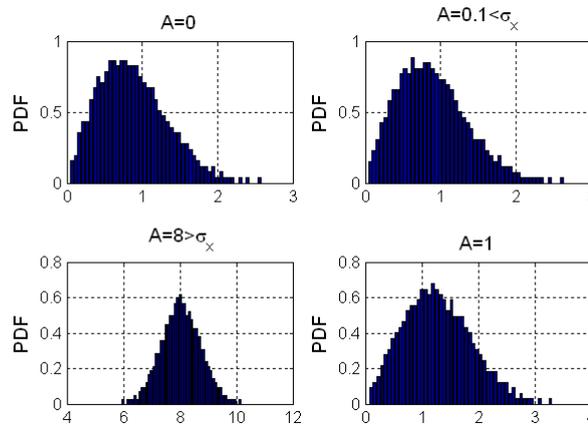


Figure 3: Rayleigh and Rice random variables.

2.4. Rayleigh and Uniform Random Variables

In this demonstration, illustrated in Figure 4, is shown how the Rayleigh RV can be obtained from the uniform RV using transform:

$$y = \sqrt{-\ln(1-x)/K}, \tag{3}$$

where K is a chosen value, $0 < K \leq 1$.

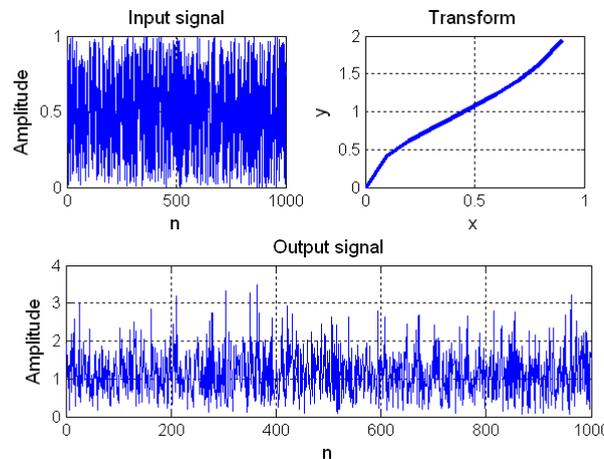


Figure 4: Rayleigh and uniform random variables, ($K=0.8$).

2.5. Rayleigh and Exponential Random Variables

This demo shows how the exponential RV, denoted as r_e , can be obtained by a square transform of a Rayleigh RV r , as illustrated in Figure 5.

$$r_e = r^2. \tag{4}$$

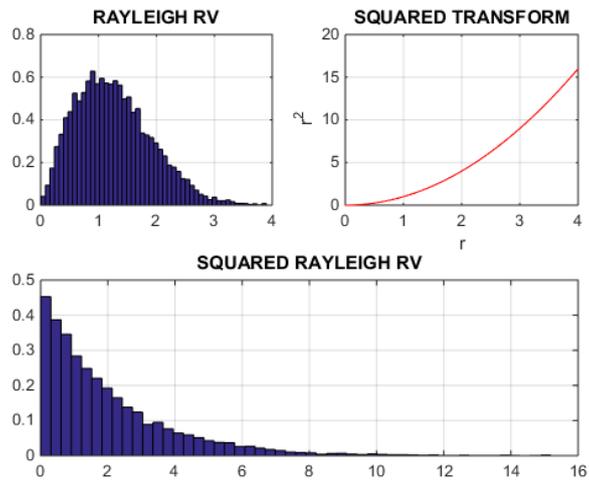


Figure 5: Rayleigh and exponential random variables.

2.6. Chi-Square and Rayleigh Random Variables

The demonstration shows how the Rayleigh RV r is related with the Chi-Square RV, r_{CS} with two degrees of freedom.

$$r_{CS} = x_1^2 + x_2^2 = r^2, \quad (5)$$

where x_1 and x_2 are zero-mean normal random variables with the equal variance $\sigma_x^2=1$.

From (5), the Rayleigh RV r is obtained from the Chi-Square RV, r_{CS} , as:

$$r = \sqrt{r_{CS}}. \quad (6)$$

Figure 6 illustrates how the Rayleigh RV is obtained from Chi Square RV by square root transformation.

3. Operations on Rayleigh Random Variable

This demo set shows some operations on Rayleigh RV, and consists of the following demonstrations:

- Linear Transform.
- Sum of m Rayleigh Random Variables.
- Product of n Rayleigh Random Variables.
- Ratio of Rayleigh Random Variables.

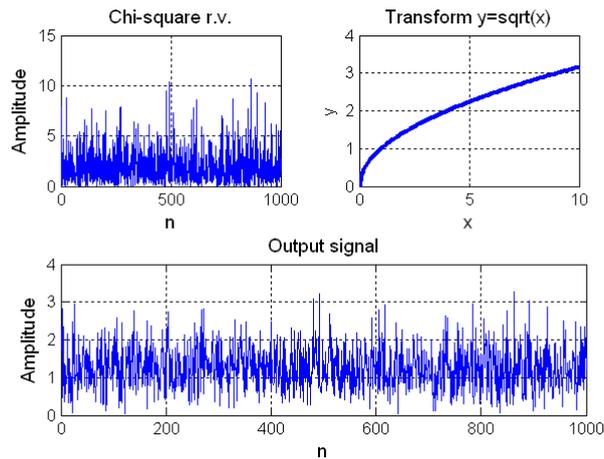


Figure 6: Chi-Square and Rayleigh random variables.

3.1. Linear Transform

In this demonstration is considered the linear transform of Rayleigh RV. The linear transform is given as $y=ax+b$, where a , and b are chosen constants. Figure 7 shows the special case ($a=2$, $b=0$), i.e. $y=2x$ and $\sigma_x^2=4$. Note that the output variable is also Rayleigh variable. However, for the negative values of the constant a , and any value of b , the values of the range of the output variable are not from 0 to infinity. Consequently, the output variable cannot be a Rayleigh variable.

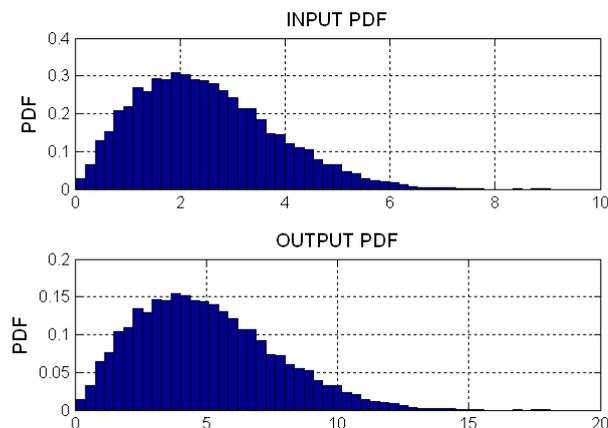


Figure 7: Special case of linear transform.

3.2. Sum of m Rayleigh Random Variables

This demonstration shows that the sum of m independent, equally distributed Rayleigh variables r_i ,

$$y = \sum_{i=1}^m r_i, \quad (7)$$

according to the Central Limit Theorem, approaches to a normal RV, as illustrated in Figure 8, for $m=5$ and the parameter of Rayleigh variable $\sigma_x^2=1$.

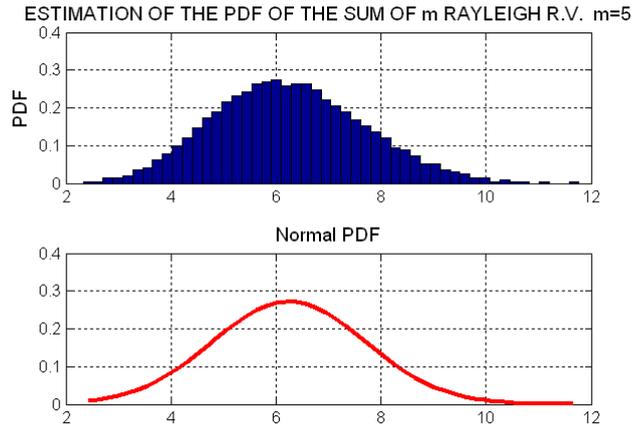


Figure 8: Sum of m independent Rayleigh RVs and normal RV.

3.3. Product of n Rayleigh Variables

The product of n statistically independent Rayleigh fading processes, called n -Rayleigh model, is introduced to describe the signal amplitude in wireless propagation (Aktas 2020; Algorhani 2020; Salo 2006; Krstic 2019):

$$y = \prod_{i=1}^n r_i. \tag{8}$$

The exact expressions for the PDF and distribution of n -Rayleigh model, are derived by using an inverse Mellin transform technique from statistics, in Salo (2006) and Krstic (2019).

In this demonstration, the product (8) is generated in n steps, for a chosen value n . The obtained product, and the estimated PDF are shown for each step. Figure 9 illustrates the case $n=3$, and $\sigma_x^2 = 1$.

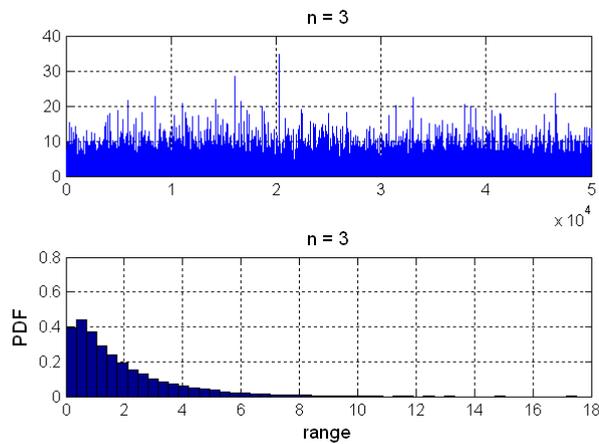


Figure 9: Product of three Rayleigh RVs and estimated PDF of the product.

3.4. Ratio of Rayleigh Random Variables

Authors in Krstic (2018) introduced two ratios, r and rr , of Rayleigh RVs to describe the second order performances of a wireless communication system, and to provide a dynamic representation of a fading channel, which has a great significance in the analysis and design of the wireless communication systems:

$$r = \frac{r_1 r_2}{r_3}; \quad rr = \frac{r_4}{r_5 r_6}, \tag{9}$$

where all Rayleigh RVs r_1, \dots, r_6 have different parameters.

In this demonstration are presented both cases. The variables r and rr are generated for chosen parameters, and the PDFs are estimated. One example is shown in Figure 10, for the Rayleigh parameters $\sigma_x^2=2$, for r_1 , $\sigma_x^2=4$ for r_2 , and $\sigma_x^2=9$, for r_3 .

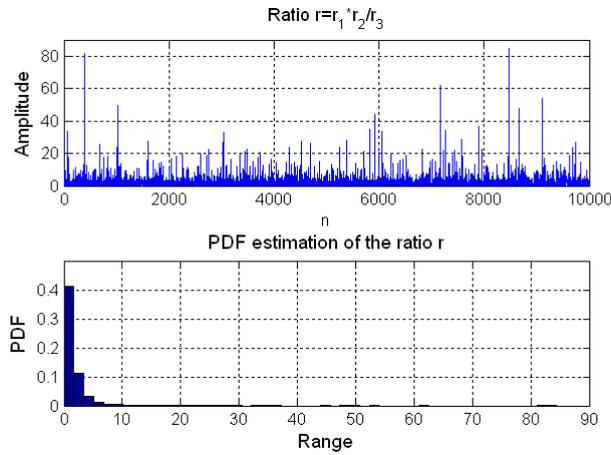


Figure 10: Ratio of Rayleigh variables.

4. Generalization

A generalization of Rayleigh RV is obtained as (Proakis 1983):

$$r = \sqrt{\sum_{i=1}^n x_i^2}, \tag{10}$$

where x_i are statistically independent zero mean normal RVs, each with the parameter σ_x^2 .

From here, r^2 is a Chi-Square RV r_{cs} with n degrees of freedom. Figure 11 illustrates demonstration for chosen values of $n=3$, and $\sigma_x^2=0.3$.

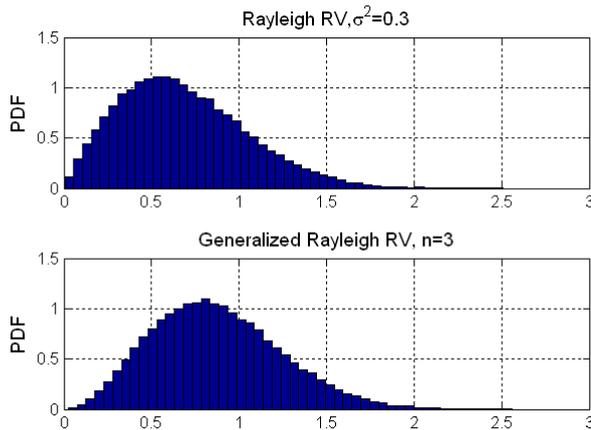


Figure 11: Rayleigh and generalized Rayleigh random variables.

5. Discussion

This demo educational tool was used as a complement to online classes during the COVID-19 pandemic for the subject of Rayleigh RV, within the curriculum Random signals and processes, at graduate level, which follows the book (Dolecek 2013). The presented features of the tool were very convenient for this unexpected online use. It is worth to mention that all our created

demo tools were also used for different subjects in the online teaching of random signals and processes. First, theoretical background was explained, followed by the corresponding set of demonstrations. The students were very active in all steps of the teaching–learning process. This tool was also useful for students in solving the homework's.

At the end of the course, a simple Quiz, about the utility of demo programs in the teaching/learning process is presented. A typical example of the Quiz can be seen in (Dolecek, 12). As expected, students were very enthusiastic with the introducing such kind of tools in teaching random variables, especially in online classes.

6. Conclusion

This education tool fulfilled the following principal goals:

- This demo tool was very useful in keeping the interest and activities of students, since it was a challenge to keep the activity and interest of students in the online classes during the COVID-19 pandemic.
- Since this important subject was not covered by our previously created demo programs, or any other tool in literature, the presented demo is filling the gap in the existing demo presentations.
- The tool provides a visualization of the behavior of this important variable, which is important in better understanding of this subject, usually given by formulas.

The students liked this way of teaching which helped them to better understand this subject.

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